KU LEUVEN

# Introduction to the Design and Cryptanalysis of Cryptographic Hash Functions

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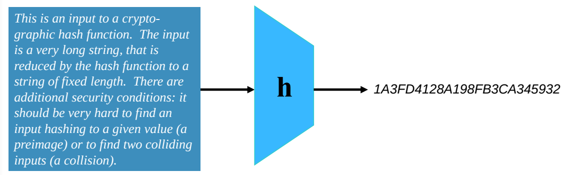
## Hash functions

X.509 Annex D
MDC-2
MD2, MD4, MD5
SHA-1

RIPEMD-160
SHA-256
SHA-512

SHA-3

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## Applications

- short unique identifier to a string
  - digital signatures
  - data authentication
- one-way function of a string
  - protection of passwords
  - micro-payments
- confirmation of knowledge/commitment
- pseudo-random string generation/key derivation
- entropy extraction
- construction of MAC algorithms, stream ciphers, block ciphers,…

2005: 800 uses of MD5 in Microsoft Windows

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## Agenda

- Definitions
- Iterations (modes)
- Compression functions
- Constructions
- SHA-3
- Conclusions

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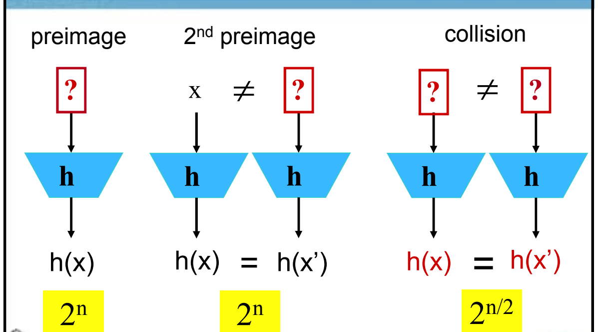
## Security requirements (n-bit result)

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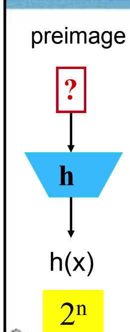
## Preimage resistance




- in a password file, one does not store
  - (username, password)
- but
  - (username,hash(password))
- this is sufficient to verify a password
- an attacker with access to the password file has to find a preimage

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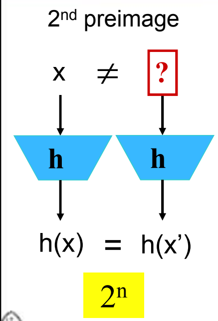
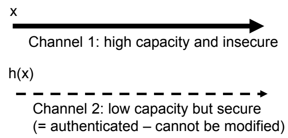
## Second preimage resistance




- an attacker can modify x but not h(x)
- he can only fool the recipient if he finds a second preimage of x

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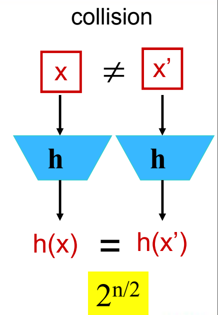
## Collision resistance

- hacker Alice prepares two versions of a software driver for the O/S company Bob
  - x is correct code
  - x' contains a backdoor that gives Alice access to the machine
- Alice submits x for inspection to Bob
- if Bob is satisfied, he digitally signs h(x) with his private key
- Alice now distributes x' to users of the O/S; these users verify the signature with Bob's public key
- this signature works for x and for x', since h(x) = h(x')

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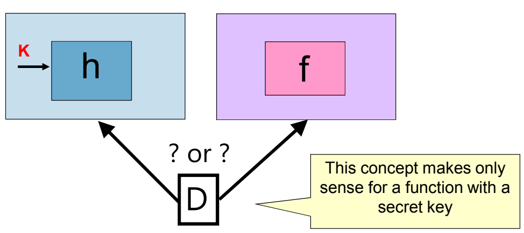
## Pseudo-random function

computationally indistinguishable from a random function

$Adv_h^{prf} = \Pr [ K \xleftarrow{\$} K: A^{h_K(.)} \Rightarrow 1] - \Pr [ f \xleftarrow{\$} RAND(m,n): A^f \Rightarrow 1]$

RAND(m,n): set of all functions from m-bit to n-bit strings

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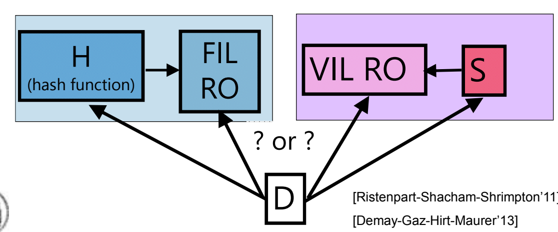
## Indifferentiability from a random oracle or PRO property [Maurer+04]

variant of indistinguishability appropriate when distinguisher has access to inner component (e.g. building block of a hash function)

∃ Simulator S, ∀ distinguisher D, $Adv^{PRO}(H,S)$ is small




[Ristenpart-Shacham-Shrimpton'11]

[Demay-Gaz-Hirt-Maurer'13]

## Brute force (2nd) preimage

- **multiple target second preimage (1 out of many):**
  - if one can attack $2^t$ simultaneous targets, the effort to find a single preimage is $2^{n-t}$
- **multiple target second preimage (many out of many):**
  - time-memory trade-off with $\Theta(2^n)$ precomputation and storage $\Theta(2^{2n/3})$ time per (2nd) preimage: $\Theta(2^{2n/3})$ [Hellman'80]
- answer: randomize hash function with a parameter S (salt, key, spice,…)

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## Brute force attacks in practice

- (2nd) preimage search
  - n = 128: 14 B$ for 1 year if one can attack $2^{40}$ targets in parallel
- parallel collision search: small memory using cycle finding algorithms (distinguished points)
  - n = 128: 1 M$ for 5 hours (or 1 year on 60K PCs)
  - n = 160: 56 M$ for 1 year
  - need 256-bit result for long term security (30 years or more)

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## Quantum computers

- in principle exponential parallelism
- inverting a one-way function: $2^n$ reduced to $2^{n/2}$ [Grover'96]
- collision search: can we do better than $2^{n/2}$ ?
  - $2^{n/3}$ computation + hardware [Brassard-Hoyer-Tapp'98] = $2^{2n/3}$
  - [Bernstein'09] classical collision search requires $2^{n/4}$ computation and hardware (= standard cost of $2^{n/2}$ )


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## Properties in practice

- collision resistance is not always necessary
- other properties are needed:
  - PRF: pseudo-randomness if keyed (with secret key)
  - PRO: pseudo-random oracle property
  - near-collision resistance
  - partial preimage resistance (most of input known)
  - multiplication freeness
- how to formalize these requirements and the relation between them?

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# Iteration
## (mode of compression function)

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## How not to construct a hash function

- Divide the message into t blocks $x_i$ of n bits each

Message block 1: $x_1$

$\oplus$

Message block 2: $x_2$

$\oplus$

...

$\oplus$

Message block t: $x_t$

=

**Hash value h(x)**

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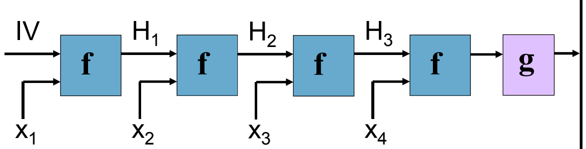
## Hash function: iterated structure




- split messages into blocks of fixed length and hash them block by block with a compression function f
- need padding at the end

efficient and elegant…. but …

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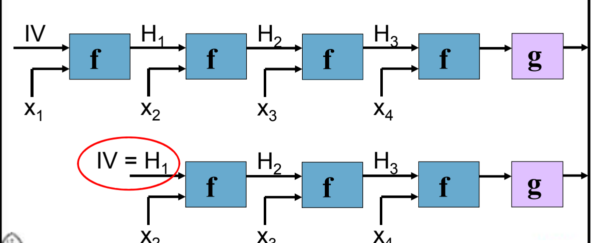
## Security relation between f and h

- iterating f can degrade its security
  - trivial example: 2nd preimage

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## Security relation between f and h (2)

- solution: Merkle-Damgård (MD) strengthening
  - fix IV, use unambiguous padding and insert length at the end
- f is collision resistant $\Rightarrow$ h is collision resistant
  [Merkle'89-Damgård'89]
- f is ideally 2nd preimage resistant $\overset{?}{\Leftrightarrow}$ h is ideally 2nd preimage resistant [Lai-Massey'92]
- PRO preservation $\Rightarrow$ Col, Sec and Pre for ideal compression function
  - but for narrow pipe bounds for Sec and Pre are at most $2^{n/2}$ rather than $2^n$
- many other results

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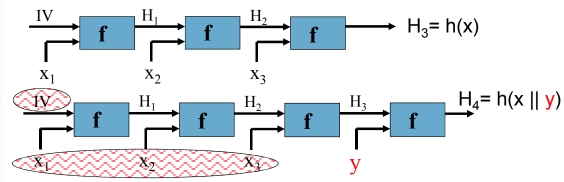
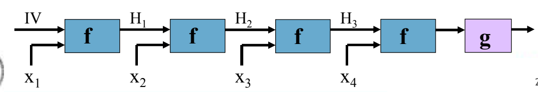
## Security relation between f and h (3)

length extension: if one knows h(x), easy to compute h(x || y) without knowing x or IV




solution: output transformation

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## Attacks on MD-type iterations

- **long message 2nd preimage attack**
  [Dean-Felten-Hu'99], [Kelsey-Schneier'05]
  - Sec security degrades lineary with number $2^t$ of message **blocks** hashed: $2^{n-t+1} + t\ 2^{n/2+1}$
  - appending the length does not help here!
- **multi-collision attack and impact on concatenation** [Joux'04]
- **herding attack** [Kelsey-Kohno'06]
  - reduces security of commitment using a hash function from $2^n$
  - on-line $2^{n-t}$ + precomputation $2.2^{(n+t)/2}$ + storage $2^t$

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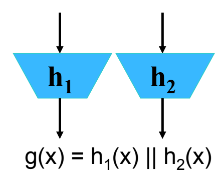
## How (NOT) to strengthen a hash function?

[Coppersmith'85][Joux'04]

- answer: concatenation
- $h_1$ (n1-bit result) and $h_2$ (n2-bit result)




- intuition: the strength of g against collision/(2nd) preimage attacks is the product of the strength of $h_1$ and $h_2$
  - if both are "independent"
- but….

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## Multiple collisions $\neq$ multi-collision

Assume "ideal" hash function h with n-bit result

- $\Theta(2^{n/2})$ evaluations of h (or steps): 1 collision
  - h(x)=h(x')
- $\Theta(r.\ 2^{n/2})$ steps: $r^2$ collisions
  - $h(x_1)=h(x_1')$ ; $h(x_2)=h(x_2')$ ; … ; $h(x_{r^2})=h(x_{r^2}')$
- $\Theta(2^{2n/3})$ steps: a 3-collision
  - h(x)= h(x')=h(x'')
- $\Theta(2^{n(t-1)/t})$ steps: a t-fold collision (multi-collision)
  - $h(x_1)= h(x_2)= \ldots =h(x_t)$

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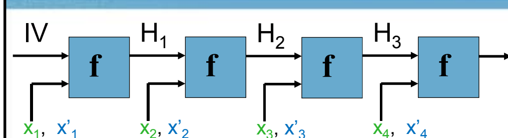
## Multi-collisions on iterated hash function (2)




- for IV: collision for block 1: $x_1$, $x'_1$
- for $H_1$: collision for block 2: $x_2$, $x'_2$
- for $H_2$: collision for block 3: $x_3$, $x'_3$
- for $H_3$: collision for block 4: $x_4$, $x'_4$
- now $h(x_1||x_2||x_3||x_4) = h(x'_1||x_2||x_3||x_4) = h(x'_1||x'_2||x_3||x_4) = \ldots$ $= h(x'_1||x'_2||x'_3||x'_4)$ **a 16-fold collision (time: 4 collisions)**

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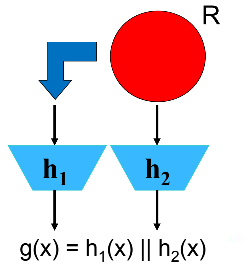
## Multi-collisions [Coppersmith'85][Joux '04]

- finding multi-collisions for an iterated hash function is not much harder than finding a single collision (if the size of the internal memory is n bits)
  - algorithm
    - generate R = $2^{n1/2}$-fold multi-collision for $h_2$
    - in R: search by brute force for $h_1$
  - Time: n1. $2^{n2/2} + 2^{n1/2} << 2^{(n1 + n2)/2}$

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## Multi-collisions [Coppersmith'85][ Joux '04]

consider $h_1$ (n1-bit result) and $h_2$ (n2-bit result), with n1 ≥ n2.

concatenation of 2 iterated hash functions ($g(x)= h_1(x) || h_2(x)$) is as most as strong as the strongest of the two (even if both are independent)

- cost of collision attack against g at most n1 . $2^{n2/2} + 2^{n1/2} << 2^{(n1 + n2)/2}$
- cost of (2nd) preimage attack against g at most n1 . $2^{n2/2} + 2^{n1} + 2^{n2} << 2^{n1 + n2}$
- if either of the functions is weak, the attacks may work better

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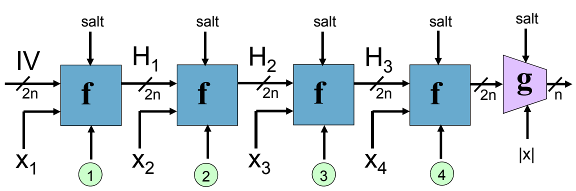
## Improving MD iteration

salt + output transformation + counter + wide pipe




security reductions well understood
many more results on property preservation
impact of theory limited

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## Improving MD iteration

- degradation with use: salting (family of functions, randomization)
  - or should a salt be part of the input?
- PRO: strong output transformation g
  - also solves length extension
- long message 2nd preimage: preclude fix points
  - counter f → $f_i$ [Biham-Dunkelman'07]
- multi-collisions, herding: avoid breakdown at $2^{n/2}$ with larger internal memory: known as wide pipe
  - e.g., extended MD4, RIPEMD, [Lucks'05]

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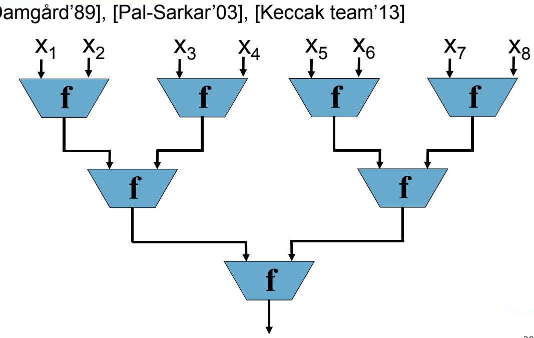
## Tree structure: parallelism

[Damgård'89], [Pal-Sarkar'03], [Keccak team'13]

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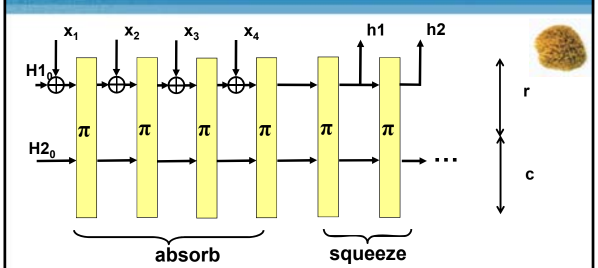
## Permutation (π) based: sponge




if result has n bits, H1 has r bits (rate), H2 has c bits (capacity) and the permutation π is "ideal"

| | |
|---|---|
| collisions | min ($2^{c/2}$ , $2^{n/2}$) |
| 2nd preimage | min ($2^{c/2}$ , $2^n$) |
| preimage | min ($2^c$ , $2^n$) |

## Modes: summary

- growing theory to reduce security properties of hash function to that of compression function (MD) or permutation (sponge)
  - preservation of large range of properties
  - relation between properties
- it is very nice to assume multiple properties of the compression function f, but unfortunately it is very hard to verify these
- still no single comprehensive theory

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# Compression functions

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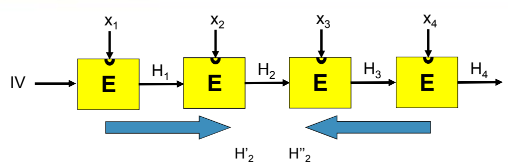
## Single block length: [Rabin'78]




- Merkle's meet-in-the-middle: ($2^{nd}$) preimage in time $2^{n/2}$
  - select $2^{n/2}$ values for $(x_1,x_2)$ and compute forward $H'_2$
  - select $2^{n/2}$ values for $(x_3,x_4)$ and compute backward $H''_2$
  - by the birthday paradox expect a match and thus a ($2^{nd}$) preimage

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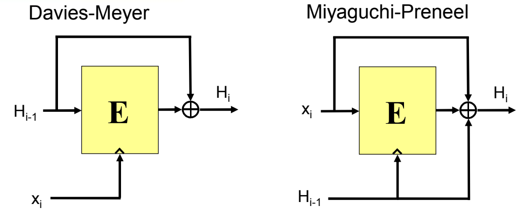
## Block cipher ($E_K$) based: single block length




- output length = block length m; rate 1; 1 key schedule per encryption
- 12 secure compression functions (in ideal cipher model)
  - lower bounds: collision $2^{m/2}$, (2nd) preimage $2^m$
- [Preneel+'93], [Black-Rogaway-Shrimpton'02], [Duo-Li'06], [Stam'09],…

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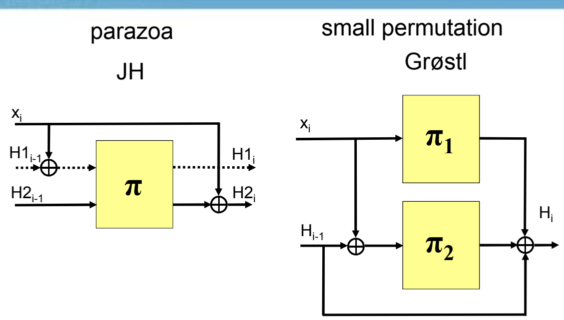
## Permutation (π) based

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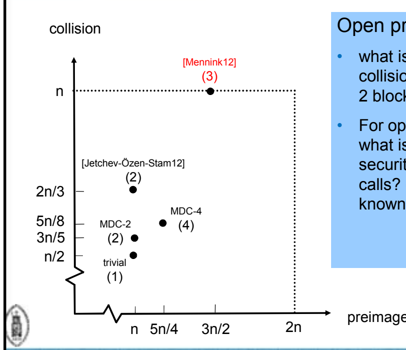
## Block cipher ($E_K$) based: double block length (3n to 2n compression)




Open problems:

- what is the best collision/preimage security for 2 block cipher calls?
- For optimal collision security: what is the best preimage security for s block cipher calls? (upper bounds are known)

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## Iteration modes and compression functions

- security of simple modes well understood
- powerful tools available
- analysis of slightly more complex schemes very difficult
- MD versus sponge debate:
  - sponge is simpler
  - should $x_i$ and $H_{i-1}$ be treated differently?

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# Hash function constructions

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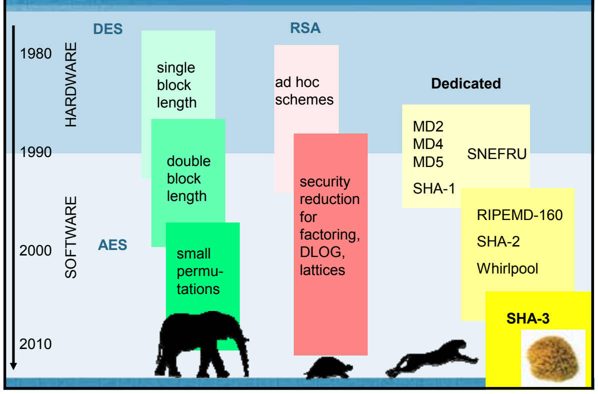
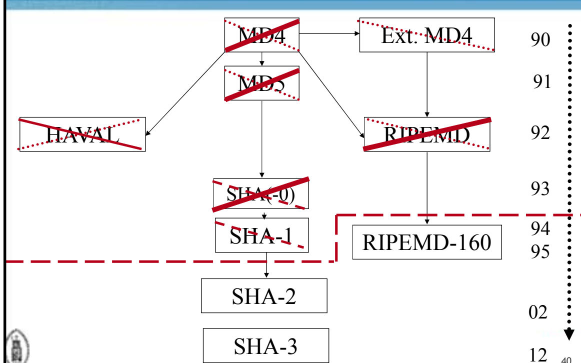
## Hash function history 101




## MDx-type hash function history

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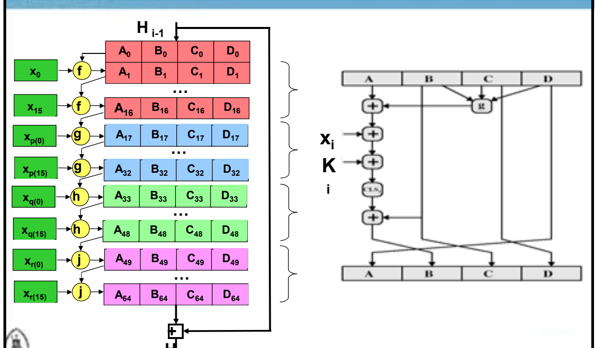
## MD5 [Rivest'91]: 4 rounds of 16 steps

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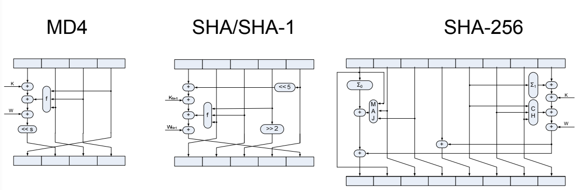
## State updates in the MD4 family




Design principles copied in MD5, RIPEMD, HAVAL, SHA, SHA-1, SHA-256, ...

- All hash functions in use today

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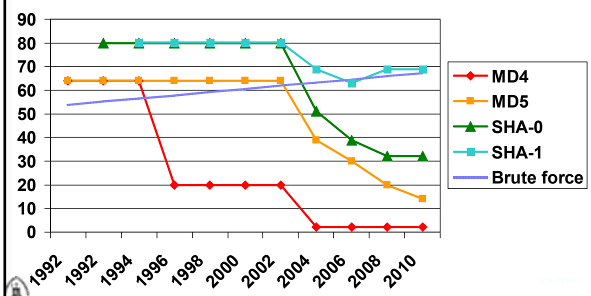
Slide credit: C. Rechberger

## The complexity of collision attacks

brute force: 1 million PCs (1 year) or US$ 100,000 hardware (4 days)

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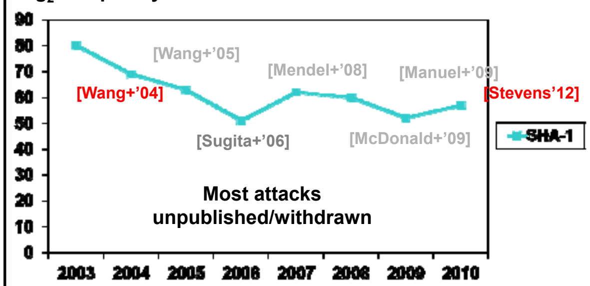
## SHA-1 designed by NIST (NSA) in '94



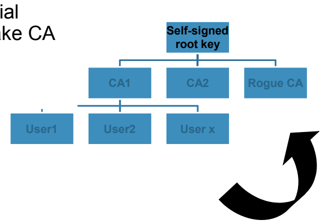

prediction: collision for SHA-1 in the next 12 months

## Rogue CA attack

[Sotirov-Stevens-Appelbaum-Lenstra-Molnar-Osvik-de Weger '08]

- request user cert; by special collision this results in a fake CA cert (need to predict serial number + validity period)

impact: **rogue CA** that can issue certs that are trusted by all browsers




- 6 CAs have issued certificates signed with MD5 in 2008:
  - Rapid SSL, Free SSL (free trial certificates offered by RapidSSL), TC TrustCenter AG, RSA Data Security, Verisign.co.jp

## Upgrades

- RIPEMD-160 is good replacement for SHA-1
- upgrading algorithms is always hard
- TLS uses MD5 || SHA-1 to protect algorithm negotiation (up to v1.1)
- **upgrading negotiation algorithm is even harder: need to upgrade TLS 1.1 to TLS 1.2**

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## SHA-2 [FIPS180,NIST'02]

- SHA-224, SHA-256, SHA-384, SHA-512
  - non-linear message expansion
  - 64/80 steps
  - SHA-384 and SHA-512: 64-bit architectures
- SHA-256 collisions: 31/64 steps $2^{65.5}$ [Mendel+'13]
  - free start collision: 52/64 steps ($2^{12x}$) [Li+12]
  - non-randomness 47/64 steps (practical) [Biryukov+11][Mendel+11]
- SHA-256 preimages: 45/64 steps ($2^{25x}$) [Khovratovitch'12]
- implementations today faster than anticipated
- adoption accelerated by other attacks on TLS
  - since 2013 deployment in TLS 1.2

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## SHA-3

(bits and bytes)

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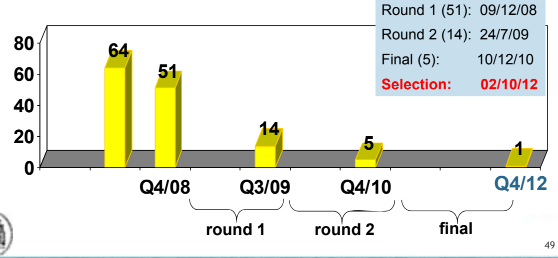
## NIST AHS competition (SHA-3)

- SHA-3: 224, 256, 384, and 512-bit message digests
- (similar to SHA-2)

| | |
|---|---|
| Call: | 02/11/07 |
| Deadline (64): | 31/10/08 |
| Round 1 (51): | 09/12/08 |
| Round 2 (14): | 24/7/09 |
| Final (5): | 10/12/10 |
| **Selection:** | **02/10/12** |

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## The candidates


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Slide credit: Christophe De Cannière

## Preliminary cryptanalysis


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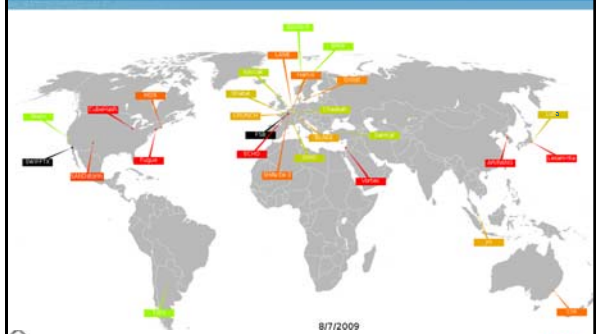
Slide credit: Christophe De Cannière

## End of Round 1 candidates

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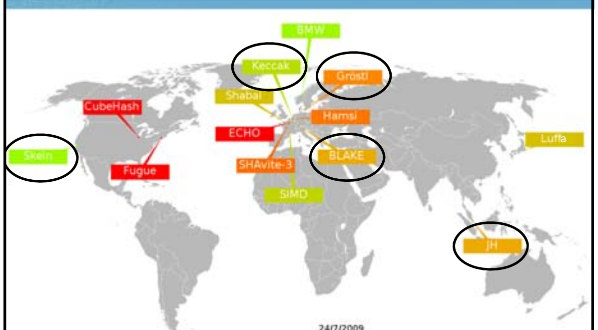
Slide credit: Christophe De Cannière

## Round 2 candidates

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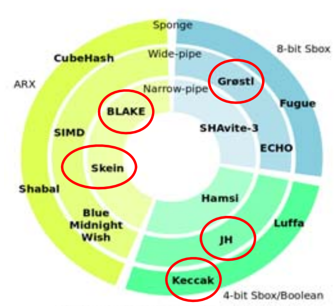
Slide credit: Christophe De Cannière

## Properties: bits and bytes

[Watanabe'10]



## Reductions: 256-bit result

| | pre | sec | coll. | indiff. | assumption |
|---|---|---|---|---|---|
| Blake-256 | 256 | 256 | 128 | 128 | E ideal |
| Grøstl-256 | 256 | 256-L | 128 | 128 | π,ρ ideal |
| JH-256 | 256 | 256 | 128 | 256 | π ideal |
| Keccak-256 | 256 | 256 | 128 | 256 | π ideal |
| Skein-256 | 256 | 256 | 128 | 256 | E ideal |
| SHAKE-128 | 128 | 128 | 128 | 128 | π ideal |
| NIST | 256 | 256-L | 128 | - | |

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## Reductions: 512-bit result

| | pre | sec | coll. | indiff. | assumption |
|---|---|---|---|---|---|
| Blake-512 | 512 | 512 | 256 | 256 | E ideal |
| Grøstl-512 | 512 | 512-L | 256 | 256 | π,ρ ideal |
| JH-512 | 256 | 256 | 256 | 256 | π ideal |
| Keccak-512 | 512 | 512 | 256 | 512 | π ideal |
| Skein-512 | 512 | 512 | 256 | 256 | E ideal |
| SHAKE-512 | 256 | 256 | 256 | 256 | π ideal |
| NIST | 512 | 512-L | 256 | - | |

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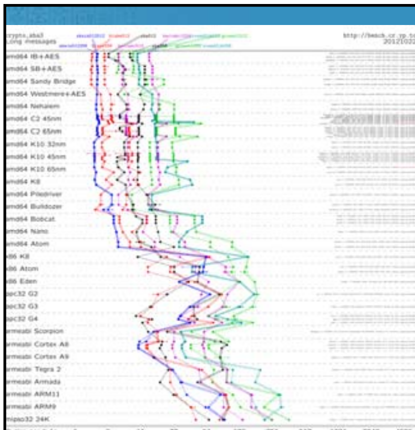
## Software performance eBash [Bernstein-Lange]



logarithmic scale

slower →

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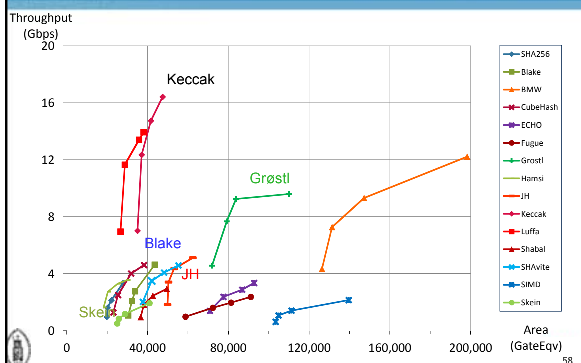
## Hardware: post-place & route results ASIC 130nm [Guo-Huang-Nazhandali-Schaumont'10]




Slide credit: Patrick Schaumont, Virginia Tech

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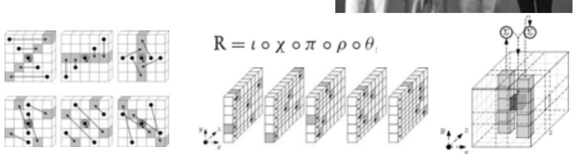
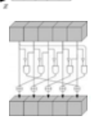
## Keccak



permutation: 25, 50, 100, 200, 400, 800, 1600

nominal version:

- 5x5 array of 64 bits
- 18 rounds of 5 steps

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## Keccak: FIPS 202 (draft: 28 May 2014)

- append 2 extra bits for domain separation to allow
  - flexible output length (XOFs or eXtendable Output Functions)
  - tree structure (Sakura) allowed by additional encoding
- 6 versions
  - SHA3-224: n=224; c = 448; r = 1152 (72%)
  - SHA3-256: n=256; c = 512; r = 1088 (68%)
  - SHA3-384: n=384; c = 768; r = 832 (52%)
  - SHA3-512: n=512; c = 1024; r = 576 (36%)
  - SHAKE128: n=x; c = 256; r = 1344 (84%)
  - SHAKE256: n=x; c = 512; r = 1088 (68%)

(SHA3-224 to SHA3-512: pad 01; SHAKE128, SHAKE256: pad 11 for XOF)

if result has n bits, H1 has r bits (rate), H2 has c bits (capacity) and the permutation π is "ideal"
- collisions: min $(2^{c/2}, 2^{n/2})$
- $2^{nd}$ preimage: min $(2^{c/2}, 2^n)$
- preimage: min $(2^c, 2^n)$

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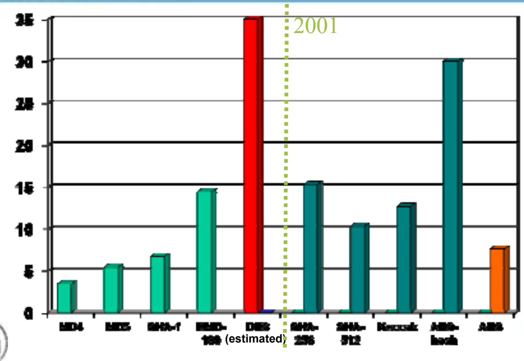
## Performance of hash functions [Bernstein-Lange]

(cycles/byte) Intel Core 2 Quad Q9550; 4 x 2833MHz (2008)

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## Hash functions: conclusions

- SHA-1 would have needed 128-160 steps instead of 80
- 2004-2009 attacks: cryptographic meltdown but not dramatic for most applications
- theory is developing for more robust iteration modes and extra features; still early for building blocks
- Nirwana: efficient hash functions with security reduction

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